A FREQUENCY ANALYSIS OF NONSTEADY HEAT TRANSFER IN A LAYER OF DISPERSE MATERIAL WITH HEAT SOURCES

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In this paper we consider the possibility of solving problems of nonsteady heat transfer in a layer by a frequency method which makes it possible, comparatively simply, to automate the calculations by means of a digital computer, skipping the stage of an analytical determination of the unknown functions in the time domain.

A gas is injected at a constant velocity v_0 in the direction h through a fixed layer of particles. Prior to entry into the layer the gas temperature is $t_g(0, \tau)$ – a function of the time τ. The heat sources in the layer represent the sum of the constant terms which are functions of the gas temperature and that of the material, as well as of the terms which are independent functions of the coordinate and time; here the material particles and the gas volume are affected by various sources of power $q_m(h, \tau)$ and $q_g(h, \tau)$, respectively:

$$q_m(h, \tau) = q_{m0} + a_g t_g(h, \tau) + a_m t_m(h, \tau) + q'_m(h, \tau); \tag{1}$$

$$q_g(h, \tau) = q_{g0} + b_g t_g(h, \tau) + b_m t_m(h, \tau) + q'_g(h, \tau). \tag{2}$$

For low values of the Bi number, when we can neglect the internal thermal resistance of the particles, we write the following system of differential heat-transfer equations:

$$\frac{\partial t_g(h, \tau)}{\partial h} + k_1 \frac{\partial t_g(h, \tau)}{\partial \tau} + k_2 [t_g(h, \tau) - t_m(h, \tau)] - k_3 [q_{g0} + b_g t_g(h, \tau) + b_m t_m(h, \tau) + q'_g(h, \tau)] = 0, \tag{3}$$

$$k_4 [t_g(h, \tau) - t_m(h, \tau)] - \frac{\partial t_m(h, \tau)}{\partial \tau} + k_5 [q_{m0} + a_g t_g(h, \tau) + a_m t_m(h, \tau) + q'_m(h, \tau)] = 0 \tag{4}$$

and we transform it after Laplace for the following zeroth initial conditions:

$$\frac{\partial t_g(h, p)}{\partial h} + k_1 p t_g(h, p) + k_2 [t_g(h, p) - t_m(h, p)] - k_3 \left[\frac{q_{g0}}{p} + b_g t_g(h, p) + b_m t_m(h, p) + q'_g(h, p)\right] = 0, \tag{3a}$$

$$k_4 [t_g(h, p) - t_m(h, p)] - p t_m(h, p)] + k_5 \left[\frac{q_{m0}}{p} + a_g t_g(h, p) + a_m t_m(h, p) + q'_m(h, p)\right] = 0. \tag{4a}$$

The solutions of systems (3a-4a) are

$$t_m(h, p) = \frac{1}{k_4 - k_5 a_m + p} \left[(k_4 + k_5 a_g) t_g(h, p) + \frac{k_5 q_{m0}}{p} + k_5 q'_m(h, p)\right] \tag{5}$$

and

$$t_g(h, p) = \exp(-Mh) \left\{t_g(0, p) + \int_0^h [N + P q'_m(h, p) + k_3 q'_g(h, p)] \exp(Mh)\, dh\right\}, \tag{6}$$

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where

$$M = k_1p + k_2 - k_3b_g - \frac{(k_2 + k_3b_m)(k_4 + k_5a_g)}{k_4 - k_5a_m + p};$$

$$N = \frac{k_5(k_2 + k_3b_m)q_{m0}}{p(k_4 - k_5a_m + p)} + \frac{k_3q_{g0}}{p};$$

$$P = \frac{k_5(k_2 + k_3b_m)}{k_4 - k_5a_m + p}.$$

In expressions (5) and (6) we can drop the term k_1p which characterizes the time required for the passage of the gas through the layer. In all cases of practical importance the coefficient k_1 is negligibly small in comparison with the remaining coefficients of the system.

The transition from the images (5) and (6) to the corresponding originals $t_g(h, \tau)$ and $t_m(h, \tau)$ is made possible by the classical methods covered in [1, 2], but the resulting expressions are not always convenient for automation of the calculations on a digital computer. To find the originals with a "computer," it is advisable to employ a frequency method for the construction of the transient responses, a method widely employed in the theory of automatic control [3].

The basis of the frequency method as it applies to the problem under consideration involves the following.

Substitution of $p = j\omega$ into (5) and (6) gives us the image of the unknown temperature in the frequency domain

$$t(h, j\omega) = \mathrm{Re}(h, \omega) + j\,\mathrm{Im}(h, \omega), \tag{7}$$

following which we find $t(h, \tau)$ from the expressions

$$t(h, \tau) = \frac{2}{\pi}\int_0^\infty \mathrm{Re}(h, \omega)\cos\tau\omega\,d\omega \tag{8}$$

or

$$t(h, \tau) = -\frac{2}{\pi}\int_0^\infty \mathrm{Im}(h, \omega)\sin\tau\omega\,d\omega \tag{9}$$

(if $\lim\limits_{\tau\to\infty} t(h, \tau) = \lim\limits_{p\to 0} pt(h, p) = 0$).

However, if

$$\lim_{\tau\to\infty} t(h, \tau) = \mathrm{const} \neq 0,$$

then

$$t(h, \tau) = \mathrm{Re}_1(h, 0) + \frac{2}{\pi}\int_0^\infty \frac{\mathrm{Im}_1(h, \omega)}{\omega}\cos\tau\omega\,d\omega \tag{10}$$

or

$$t(h, \tau) = \frac{2}{\pi}\int_0^\infty \frac{\mathrm{Re}_1(h, \omega)}{\omega}\sin\tau\omega\,d\omega, \tag{11}$$

where $\mathrm{Re}_1(h, \omega)$ and $\mathrm{Im}_1(h, \omega)$ are determined from the expressions

$$\mathrm{Re}_1(h, \omega) + j\,\mathrm{Im}_1(h, \omega) = j\omega t(h, j\omega). \tag{12}$$

The determination of the originals with expressions (8)-(11) can be accomplished on any class of digital computer. The generalized real Re and imaginary Im frequency characteristics usually attenuate rapidly with increasing frequency, so that the bounds of the improper integrals can be calculated on the basis of the rectangle formula in the interval of real frequencies

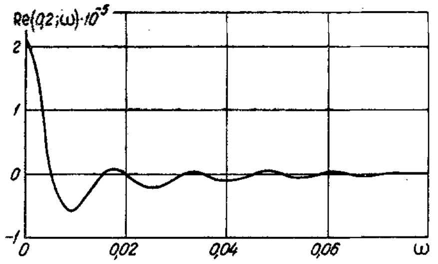

Fig. 1

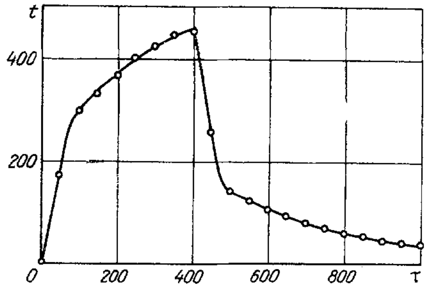

Fig. 2

Fig. 1. Real frequency characteristic of the gas temperature: ω) angular frequency, rad/sec; $\mathrm{Re}(0.2, \omega)$ the real characteristic, deg/sec.

Fig. 2. Gas temperature at the outlet from the layer: τ) time, sec; $t_g(0.2, \tau)$ the gas temperature, °C.

$$0 \leqslant \omega \leqslant \omega_{out};$$

ω_{out} is the outlet frequency at which $|\omega_{out}\mathrm{Re}(h, \omega_{out})|$ or $|\omega_{out}\mathrm{Im}(h, \omega_{out}|$ attain 0.1 of their maximum absolute values and subsequently do not exceed these quantities.

The use of the frequency method is illustrated in an example in which the solution has been achieved on a "Promin" digital computer.

Example. Determine the gas temperature at the outlet from a layer, if the zone of heat liberation – with the dimension Δh – shifts at a constant velocity v from the inlet cross section of the layer to the outlet cross section. The gas temperature in front of the inlet to the layer is given by $t_g(0, \tau)$. The power of the heat sources in the heat-liberation zone is constant:

$$q'_{m(g)}(h, \tau) = q'_{m(g)} = \mathrm{const} \quad \text{for} \quad \frac{h}{v} < \tau < \frac{h+\Delta h}{v}. \tag{13}$$

We transform (13) according to Laplace:

$$q'_{m(g)}(h, p) = \frac{q'_{m(g)}}{p}\left[\exp\left(-\frac{h}{v}p\right) - \exp\left(-\frac{h+\Delta h}{v}p\right)\right],$$

so that from (6) we obtain

$$t_g(h, p) = \frac{1}{p\left(\dfrac{k_2 p}{k_4+p} - \dfrac{p}{v}\right)}\left(\frac{k_2 k_5}{k_4+p}q'_m + k_3 q'_g\right)$$
$$\times\left[1 - \exp\left(-\frac{\Delta h}{v}p\right)\right]\left[\exp\left(-\frac{h}{v}p\right) - \exp\left(-\frac{k_2 h}{k_4+p}p\right)\right]. \tag{14}$$

In (14), if we substitute $p = j\omega$, $q'_m = 8000$ W/kg, $q'_g = 12{,}000$ W/kg, $v = 0.0005$ m/sec, $h = 0.2$ m; $\Delta h = 0.04$ m; $c_g = 1200$ J/(kg·deg), $c_m = 625$ J/(kg·deg), $\rho_g = 1.25$ kg/m³, $\rho_m = 1600$ kg/m³, $\alpha_v = 3000$ W/(m³·deg), $v_0 = 1.6$ m/sec, i.e., $k_2 = 1.25$, $k_3 = 0.66667$, $k_4 = 0.003$, and $k_5 = 0.0016$, we find

$$\mathrm{Re}(0,\ 2;\ \omega) = \frac{5\cdot 10^{-4}(ADF - BCF + ACE + BDE)}{\omega^2(5.6406\cdot 10^{-6} + \omega^2)}, \tag{15}$$

where

$$A = 0.095 + 8000\omega^2;$$
$$B = 21\omega;$$
$$C = 1 - \cos 80\omega;$$
$$D = \sin 80\omega;$$
$$E = \cos 400\omega - \exp\left(-\frac{0.25\omega^2}{9\cdot 10^{-6} + \omega^2}\right)\cos\frac{7.5\cdot 10^{-4}\omega}{9\cdot 10^{-6} + \omega^2};$$

$$F = \sin 400\omega - \exp\left(-\frac{0.25\omega^2}{9\cdot 10^{-6} + \omega^2}\right) \sin \frac{7.5\cdot 10^{-4}\omega}{9\cdot 10^{-6} + \omega^2}.$$

The curve for the real characteristic Re(0.2, ω) is shown in Fig. 1. The domain of real frequencies is bounded by the outlet ω_{out} = 0.07 rad/sec.

Since

$$\lim_{\tau \to \infty} t(0.2;\ \tau) = 0,$$

we use expression (8), transforming it to the form

$$t(0.2;\ \tau) = \frac{2}{\pi} \Delta\omega \sum_{1}^{n} \mathrm{Re}(0.2;\ \omega_n) \cos \tau\omega_n, \tag{16}$$

where

$$\Delta\omega = \frac{\omega_{out}}{n} = \frac{0.07}{350} = 0.0002 \text{ rad/sec};$$

$$\omega_1 = 0.0001 \text{ rad/sec};\ \ \omega_n = \omega_{n-1} + \Delta\omega.$$

The results of the calculations according to expression (16) are shown in Fig. 2.

NOTATION

τ	is the time;
h	is the coordinate of the layer in the direction of gas-flow motion;
t_g, t_m	are, respectively, the temperatures of the gas and the material;
q_{m0}, q_{g0}, a_g, a_m, b_g, b_m	are constants;

$k_1 = f/v_0S$, $k_2 = \alpha_v/c_g\rho_g v_0$, $k_3 = \rho_m/c_g\rho_g v_0$, $k_4 = \alpha_v/c_m\rho_m$, $k_5 = 1/c_m$;

f	is the flowthrough cross section for the gas;
S	is the lateral cross section of the layer;
v_0	is the gas velocity;
α_v	is the volumetric heat-transfer coefficient;
c_g, ρ_g	are, respectively, the heat capacity and density of the gas;
c_m, ρ_m	are, respectively, the heat capacity and bulk density of the material;
p	is the Laplace transform parameter;
ω	is the angular frequency;
$j = \sqrt{-1}$, Re(h, ω), Im(h, ω)	are the generalized real and imaginary frequency characteristics.

LITERATURE CITED

1. G. D. Rabinovich, Inzh. Fiz. Zh., 3, No. 4 (1960).
2. A. V. Lykov and Yu. A. Mikhailov, The Theory of Heat and Mass Transfer [in Russian], Gosénergoizdat (1963), p. 338.
3. V. V. Solodovnikov, Yu. I. Topcheev, and G. V. Krutikova, The Frequency Method of Constructing Transient Responses [in Russian], Gostekhteorizdat (1955), p. 12.